

Application of the New Force-Depending Radiation Reaction Approach to the Motion of a Charge in an Undulator Device

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Abstract

The effect of force-depending radiation reaction on charge motion traveling inside an undulator is studied using the new force approach for radiation reaction. The effect on the dynamics of a charged particle is determined with the hope that this one can be measured experimentally and can be determined whether or not this approach points on the right direction to understand the nature of radiation reaction.

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1 Introduction

The incorporation of radiation reaction due to radiation emission of electromagnetic waves on the equation of motion of an accelerated particle [1,2] has been a hard topic for more than two centuries. The approaches given by Abraham-Lorentz-Dirac [3,4,5] to this phenomenon resulted unsatisfactory when these are applied to observed experimental phenomenon of radiation since, for zero external forces on the charged particle, still there exist some, so called, pre-acceleration of the charged particle [6,7] which has not fundamental experimental verification. The experimental fact that is needed to take into account is that radiation of the charged particle disappears as soon as the acceleration of the particle disappears, and this acceleration disappears as soon as the external force becomes zero. This implies that radiation reaction force (damping force associated to emission of electromagnetic waves) must be a function of the external force. This observation was pointed out in [8], and the new proposition made was that radiation reaction force must be a function of the external force explicitly, and a modified relativistic expression for the motion of a charged particle was given there. In this paper, we apply this approach to the dynamical motion of a relativistic single charged particle traveling in an undulator, and we make the comparison of the dynamical motion with and without radiation reaction force, with the idea of to see and to quantify a possible difference which could be measured to verified experimentally this new proposition for the radiation reaction.

2 Equation of Motion

Consider the motion of a single charged particle inside an undulator magnet [9] of length L with a magnetic field given by

$$\mathbf{B}(y) = (0, 0, B_0 \sin \frac{2\pi y}{\lambda_u}), \tag{1}$$

where λ_u is the wave length of the undulator, defined by the periodicity of the ferromagnetic elements of the undulator (magnetic elements with alternating N-S pole arranged), and having its symmetry along the y -axis. The external force on the charged particle is (CGS units) [1]

$$\mathbf{F} = \frac{q}{c} \mathbf{v} \times \mathbf{B}, \tag{2}$$

where q and \mathbf{v} are the charge and the velocity of the particle, and c is the speed of light. The radiation reaction force term is given by [10]

$$\mathbf{F}_{rad} = -\frac{q^2 |\aleph \mathbf{F}|^2 \mathbf{v}}{4\pi m^2 c^3 \gamma^2 v^2} \int_{\Omega} \frac{|\mathbf{e}_1 \sin \theta_1 - \mathbf{e}_2 \beta \sin \theta_2|^2}{1 - \beta \cos \theta} d\Omega, \tag{3}$$

where m , v , β are the mass, the speed, and the normalized speed ($\beta = v/c$) of the charged particle, γ is the usual relativistic factor ($\gamma = (1 - \beta^2)^{-1/2}$), and \aleph is a matrix defined as

$$\aleph = \begin{pmatrix} 1 - \beta_x^2 & -\beta_x \beta_y & -\beta_x \beta_z \\ -\beta_y \beta_x & 1 - \beta_y^2 & -\beta_y \beta_z \\ -\beta_z \beta_x & -\beta_z \beta_y & 1 - \beta_z^2 \end{pmatrix}, \tag{4}$$

with β_i for $i = x, y, z$ being the normalized components of the velocity of the charged particle. Since the problem is similar to a circular motion, the radiation reaction force is just

$$\mathbf{F}_{rad} = -\frac{\lambda_0 F^2}{v^2 \gamma^2} \mathbf{v}, \tag{5}$$

where λ_0 is defined as

$$\lambda_0 = \frac{2q^2}{3m^2 c^3}. \tag{6}$$

Therefore, the relativistic equation of motion is

$$\frac{d(\gamma \mathbf{v})}{dt} = \mathbf{F} - \frac{\lambda_0 F^2}{v^2 \gamma^2} \mathbf{v}. \quad (7)$$

In terms of the variable $\vec{\beta} = \mathbf{v}/c$ and after making the differentiation of γ , the equation for $\vec{\beta}$ is

$$\dot{\vec{\beta}} = \frac{1}{mc\gamma} \mathfrak{R}\mathbf{F} - \frac{\lambda_0 F^2}{mc^2 \beta^2 \gamma^3} \mathfrak{R}\vec{\beta}. \quad (8)$$

Substituting (1) and (2) in the above expression and after some rearrangements, one gets the following dynamical system

$$\dot{x} = \beta_x c \quad (9a)$$

$$\dot{\beta}_x = A\beta_y \cos \frac{2\pi y}{\lambda_u} - B \left[(1 - \beta_x^2)\beta_x - \beta_x \beta_y^2 - \beta_x \beta_z^2 \right] \quad (9b)$$

$$\dot{y} = \beta_y c \quad (10a)$$

$$\dot{\beta}_y = -A\beta_x \cos \frac{2\pi y}{\lambda_u} - B \left[-\beta_y \beta_x^2 + (1 - \beta_y^2)\beta_y - \beta_y \beta_z^2 \right] \quad (10b)$$

$$\dot{z} = \beta_z c \quad (11a)$$

$$\dot{\beta}_z = -B \left[-\beta_z \beta_x^2 - \beta_z \beta_y^2 + (1 - \beta_z^2)\beta_z \right], \quad (11b)$$

where the constants A and B are defined as

$$A = \frac{cqB_0}{mc^2 \gamma}, \quad \text{and} \quad B = \frac{\lambda_0 F^2}{mc^2 \beta^2 \gamma^3}, \quad (12)$$

and the magnitude of the force is

$$F = qB_0 \sqrt{\beta_x^2 + \beta_y^2} \cos \frac{2\pi y}{\lambda_u}. \quad (13)$$

This dynamical systems is defined in the space $\Omega = \mathfrak{R}^3 \times [-1, 1]^3$, and the critical points in this space is the set points $\{(\mathbf{x}, \vec{\beta}) \in \Omega \mid \vec{\beta} = \vec{0}\}$. The linear matrix around each critical point is

$$D_{\mathbf{x}} = \begin{pmatrix} 0 & 0 & 0 & c & 0 & 0 \\ 0 & 0 & 0 & 0 & c & 0 \\ 0 & 0 & 0 & 0 & 0 & c \\ 0 & -a & 0 & -b & 0 & 0 \\ 0 & -a & 0 & 0 & -b & 0 \\ 0 & -a & 0 & 0 & 0 & -b \end{pmatrix}, \quad (14)$$

where a and b are given by

$$a = -\frac{2A\pi}{\lambda_u} \sin \frac{2\pi y}{\lambda_u} - \frac{2\lambda_0 q^2 B_0^2}{mc^2} \cos(2\pi y/\lambda_u) \sin(2\pi y/\lambda_u) \quad (15a)$$

and

$$b = \lambda_0 q^2 B_0^2 \cos^2(2\pi y/\lambda_u)/mc^2. \quad (15b)$$

The set of eigenvalues of this matrix, $\{\lambda_i\}_{i=1,\dots,6}$ are such that $Re(\lambda_i) \leq 0$ for $i = 1, \dots, 6$ (in fact, one has that $Re(\lambda_i) \sim -b$ or $Re(\lambda_i) \sim -b \pm \sqrt{b^2 - 4ac}$). Therefore, our dynamical system is stable for any $\mathbf{x} \in \mathfrak{R}^3$, and this stability is due to radiation reaction force, as one could have expected.

3 Results

Using a typical parameters for a free electron laser (FEL)

$$\lambda_u = 4 \text{ cm}, \quad B_0 = 2000 \text{ Gauss}, \quad q_e = -4.803 \times 10^{-10} \text{ esu}, \quad m_e = 9.109 \times 10^{-34} \text{ gr}, \quad c \approx 3 \times 10^{10} \text{ cm/s}, \quad (16)$$

and the initial conditions of the electron as

$$x(0) = y(0) = z(0) = 0, \quad \beta_x(0) = \beta_z(0) = 0.00001, \quad \beta_y(0) = 0.99, \quad (17)$$

the figure below shows the difference (without and with radiation reaction force) of the normalized components of the velocity of the electron as a function of the length of the undulator (L). The continuous curve is the analytical fix to the behavior of this difference, which is of the form

$$\Delta\beta_i = a_i L^2 e^{b_i L}, \quad i = x, y, \quad (18)$$

where $a_x = 1.94268 \times 10^{-16} \text{ cm}^{-2}$, $a_y = 2.50485 \times 10^{-17} \text{ cm}^{-2}$, $b_x = 1.40958 \times 10^{-05} \text{ cm}^{-1}$, and $b_y = 2.26834 \times 10^{-06} \text{ cm}^{-1}$.

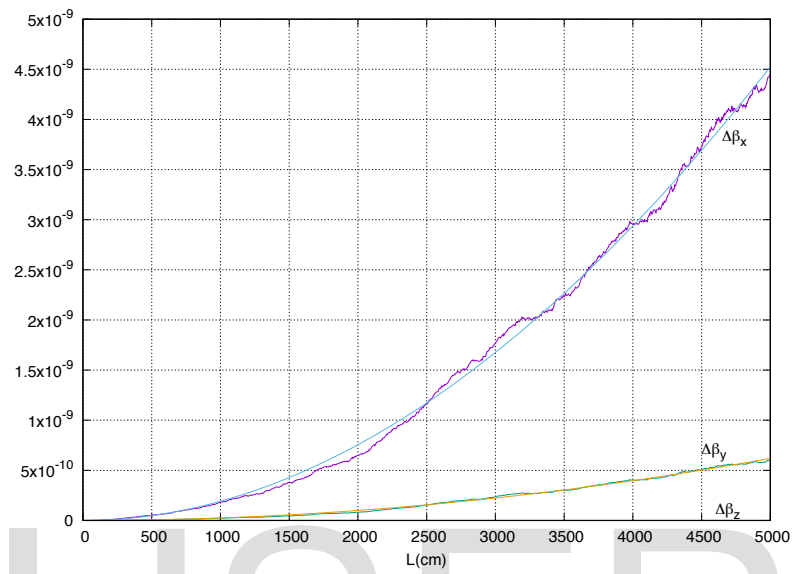


Figure 1: Difference of the normalized velocity components as a function of the undulator length.

4 Conclusions

The force-depending radiation reaction approach was used to study the dynamical motion of a single charged particle in an undulator device. We have shown that there is a small difference in the dynamics of the particle when radiation reaction force is taken into account, and this difference increases exponentially as a function of the undulator length L . Although, the difference on velocity components is quite small even for long undulators, the results suggest that this difference could be measured experimentally, and in turns, one can see whether or not this approach points in the right direction to understand radiation reaction force.

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